

## CALCULATION POLICY

Towngate Primary Academy

Subject Coordinator: Mrs A Bateman



This calculation policy has been developed and agreed after using the National Curriculum objectives and methods in conjunction to the Maths Hub schemes of learning. This calculation policy sets out the methods that children will be taught and encouraged to use when tackling calculations as part of their daily Maths lessons and during their work in a wide range of cross-curricular and real-life contexts. The policy reflects our belief that the methods taught should make sense to children and should be both efficient and reliable.

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

#### What makes a Mathematician?

A definition of a Mathematician:

- Someone who can see patterns;
- Someone who shows deeper application;
- Someone who identifies and understands the interconnectivity of concepts and demonstrates this through the transfer of skills;
- Someone who is systematic and resilient and can extend their own learning.

At Towngate Primary Academy, we believe that in order to develop confident, competent and resilient young mathematicians, our pupils benefit from following a mathematical journey to further explore and discover mathematical concepts. To ensure consistency across school, our pupils' learning is deepened by following a clear sequence of learning opportunities:

fluency reasoning problem solving



#### Fluency

One of the three aims of the National Curriculum states that pupils (of all ages, not just primary children) will:

"become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately."

Fluency is more than simply practicing a calculation or following a procedure; fluency relies on demonstrating an efficient approach to mathematics; choosing to use efficient strategy that can be carried out easily and quickly. Fluency demands accuracy and sound understanding of key number facts, concepts and relationships whilst relying on flexibility to understand that maths can be presented in a number of different ways. Fluency demands more of pupils than simply memorising a single procedure – they need to understand why they are doing what they are doing and know when it is appropriate to use different methods.

A key element of fluency is procedural variation: to be confident mathematicians, pupils need to see maths in a variety of different ways – using key concepts to apply what they already know to new learning.

#### Reasoning

The second aim of the National Curriculum indicates that pupils will:

"reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language."

Effective reasoning relies on using and applying prior knowledge to a given context or problem. Reasoning requires logical thinking to tackle a challenging concept – allowing time to try different strategies to reach a given outcome. In practice, through reasoning focused lessons, pupils develop their ability to select appropriate strategies to solve a problem, to draw logical conclusions, to develop and explain solutions / methods and to reflect on their own mathematic challenges and successes. In order for our pupils to reason accurately, we use a range of different question stems to promote discussion and discovery. Some of these include: spot the mistake, find the odd one out, what's the same – what's different, convince me etc. When learners are able to explain and justify their learning and mathematical choices, these skills can be transferred into isolated problem solving investigations and opportunities.

#### Problem Solving

The third aim of the National Curriculum programme of study for mathematics explains that pupils can:

"can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions."

Problem solving allows opportunities for pupils to apply their understanding to isolated concepts. Through effective problem solving, pupils are able to make sense of mathematics through exploring 'real world' contextual problems; by applying knowledge of a given skill, pupils are able to gain a better understanding of the modern world around them. Problem solving allows for pupils to seek solutions, explore patterns, formulate conjectures.

Through problem solving, pupils learn to take risks in their learning, to persevere in a task and experience success.



#### **KEY STAGE 1**

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value.

Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 - 3 and 15 - 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.

In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.

**Multiplication and division:** Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division.

They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation.

In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.

**Fractions:** In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole.

In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.



Year 1				
	Concrete	Pictorial	Abstruct	
Year 1 Addition	Counting and adding more Children add one more person or object to a group to find one more.	Counting and adding more Children add one more cube or counter to a group to represent one more.	Counting and adding more  Use a number line to understand how to link counting on with finding one more.	
		\$\frac{1}{2} \text{\$\frac{1}{2}} \$\frac{	one more 0 1 2 3 4 5 6 7 8 9 10	
		99	One more than 6 is 7. 7 is one more than 6.	
		One more than 4 is 5.	Learn to link counting on with adding more than one. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	Understanding part-part-whole relationship (partitioning)  Sort people and objects into parts and understand the relationship with the whole.  The parts are 2 and 4. The whole is 6.	Understanding part-part-whole relationship (partitioning) Children draw to represent the parts and understand the relationship with the whole.  The parts are 1 and 5. The whole is 6.	Understanding part-part-whole relationship (partitioning) Use a part-whole model to represent the numbers. $ \begin{array}{c}                                     $	
	Knowing and finding number bonds within 10 (commutativity)  Break apart a group and put back together to find and form number bonds. $6 + 4 = 10$	Knowing and finding number bonds within 10 (commutativity) Use five and ten frames to represent key number bonds. $5 = 4 + 1$ $10 = 7 + 3$	Knowing and finding number bonds within 10 (commutativity)  Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero. $4 + 0 = 4$ $3 + 1 = 4$	



### Understanding teen numbers as a complete 10 and some more

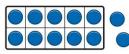
Complete a group of 10 objects and count more.

11 is 10 and 1 more.



### Understanding teen numbers as a complete 10 and some more

Use a ten frame to support understanding of a complete 10 for teen numbers.



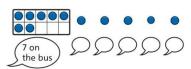
13 is 10 and 3 more.

## Understanding teen numbers as a complete 10 and some more.

1 ten and 3 ones equal 13. 10 + 3 = 13

#### Adding by counting on (augmentation)

Children use counters to support and represent their counting on strategy.



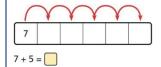
#### Adding by counting on (augmentation)

Children use knowledge of counting to 20 to find a total by counting on using people or objects.



#### Adding by counting on (augmentation)

Children use number lines or number tracks to support their counting on strategy.



#### Adding the 1s

Children use concrete materials to recognise how to add the 1s to find the total efficiently.



2 + 3 = 512 + 3 = 15

#### Adding the 1s

Children represent calculations using ten frames to add a teen and 1s.



12 + 3 = 15

2 + 3 = 5

### Adding the 1s

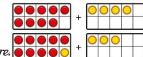
Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently.

$$3 + 5 = 8$$
  
 $S\sigma$ ,  $13 + 5 = 18$ 

#### Bridging the 10 using number bonds

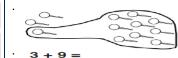
Children use counters to complete a ten frame and understand how they can add using knowledge of number honds to 10

9 add 1 makes 10. So, 9 add 4 is 10 and 3 more.



#### Bridging the 10 using number bonds

Use pictures or a number line. Regroup or partition the smaller number to make 10.



#### Bridging the 10 using number bonds

Use a part-whole model and a number line to support the calculation. 4

9 + 4 = 13



#### Year 1 Subtraction

#### Counting back and taking away (reduction)

Children arrange objects and remove to find how many are left.



1 less than 6 is 5. 6 subtract 1 is 5.

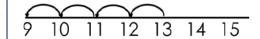
#### Counting back and taking away (reduction)

Children draw and cross out or use counters to represent objects from a problem.



#### Counting back and taking away (reduction)

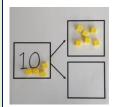
Children count back to take away and use a number line or number track to support the method.



9 - 3 = 6

## Finding a missing part, given a whole and a part (partitioning)

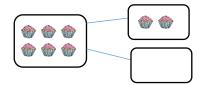
Children separate a whole into parts and understand how one part can be found by subtraction.



10 - 6 = ?

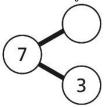
## Finding a missing part, given a whole and a part (partitioning)

Children represent a whole and a part and understand how to find the missing part by subtraction.



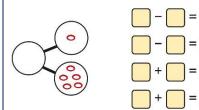
## Finding a missing part, given a whole and a part (partitioning)

Children use a part-whole model to support the subtraction to find a missing part.



7 - 3 = ?

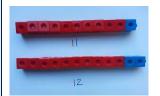
Children develop an understanding of the relationship between addition and subtraction facts in a partwhole model.





#### Finding the difference

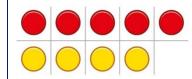
Arrange two groups so that the difference between the groups can be worked out.



12 is 1 more than 11. 11 is 1 less than 12. The difference between 11 and 12 is 1.

#### Finding the difference

Represent objects using sketches or counters to support finding the difference.

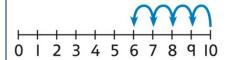


5 - 4 = 1

The difference between 5 and 4 is 1.

#### Finding the difference

Children understand 'find the difference' as subtraction.



10 - 4 = 6

The difference between 10 and 6 is 4.

#### Subtraction within 20

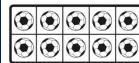
Understand when and how to subtract 1s efficiently.

Use a bead string to subtract 1s efficiently.



5 - 3 = 2 15 - 3 = 12

#### Subtraction within 20



5 - 3 = 2 15 - 3 = 12

Understand when and how to subtract 1s efficiently.



Subtraction within 20

Understand how to use knowledge of bonds within 10 to subtract efficiently.

#### Subtracting 10s and 1s

For example: 18 - 12

Subtract 12 by first subtracting the 10, then the remaining 2.





First subtract the 10, then take away 2.

#### Subtracting 10s and 1s

For example: 18 - 12



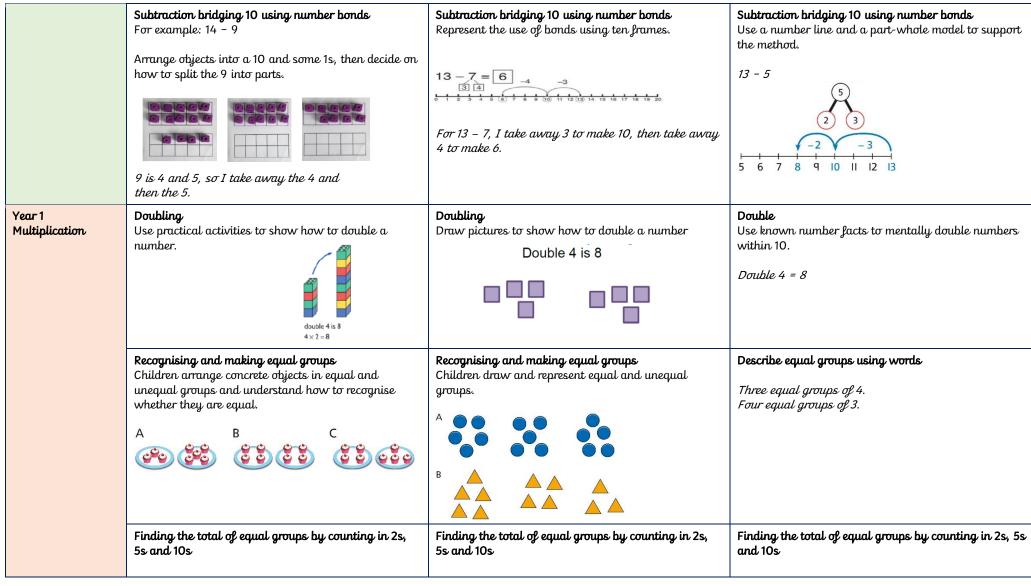
First subtract the 10, then subtract 2.

#### Subtracting 10s and 1s

Use a part-whole model to support the calculation.







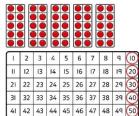




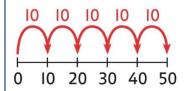
There are 5 pens in each pack ... 5...10...15...20...25...30...35...40...



100 squares and ten frames support counting in 2s, 5s and 10s.



Use a number line to support repeated addition through counting in 2s, 5s and 10s.



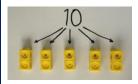
#### Year 1 Division

#### Grouping

Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.

Sort a whole set people and objects into equal groups.

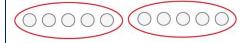




There are 10 children altogether. There are 2 in each group. There are 5 groups.

#### Grouping

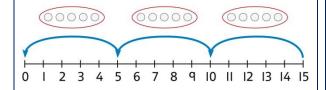
Represent a whole and work out how many equal groups.



There are 10 in total. There are 5 in each group. There are 2 groups.

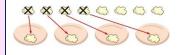
#### Grouping

Children may relate this to counting back in steps of 2, 5 or 10.



#### Sharing

Share a set of objects into equal parts and work out how many are in each part.





#### Sharing

Sketch or draw to represent sharing into equal parts. This may be related to fractions.





#### Sharing

10 shared into 2 equal groups gives 5 in each group.

$$10 \div 2 = 5$$



	Year 2			
	Concrete	Pictorial	Abstract	
Year 2 Addition				
Understanding 10s and 1s	Group objects into 10s and 1s.  Bundle straws to understand unitising of 10s.	Understand 10s and 1s equipment, and link with visual representations on ten frames.	Represent numbers on a place value grid, using equipment or numerals.  Tens Ones  3 2  Tens Ones 4 3	
Adding 10s	Use known bonds and unitising to add 10s.  I know that $4 + 3 = 7$ .  So, I know that 4 tens add 3 tens is 7 tens.	Use known bonds and unitising to add 10s.	Use known bonds and unitising to add 10s.  4 + 3 = 7 4 tens + 3 tens = 7 tens 40 + 30 = 70  4 + 3 =	
Adding a 1-digit number to a 2-digit number not bridging a 10	Aggregation and Augmentation Add the 1s to find the total. Use known bonds within 10.  +	Aggregation and Augmentation Add the 1s.  41 is 4 tens and 1 additional ones. 1 ones and 6 ones are 7 ones. The total is 3 tens and 7 additional ones.	Aggregation and Augmentation Add the 1s.  Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.  30 31 32 33 34 35 36 37 38 39 40	

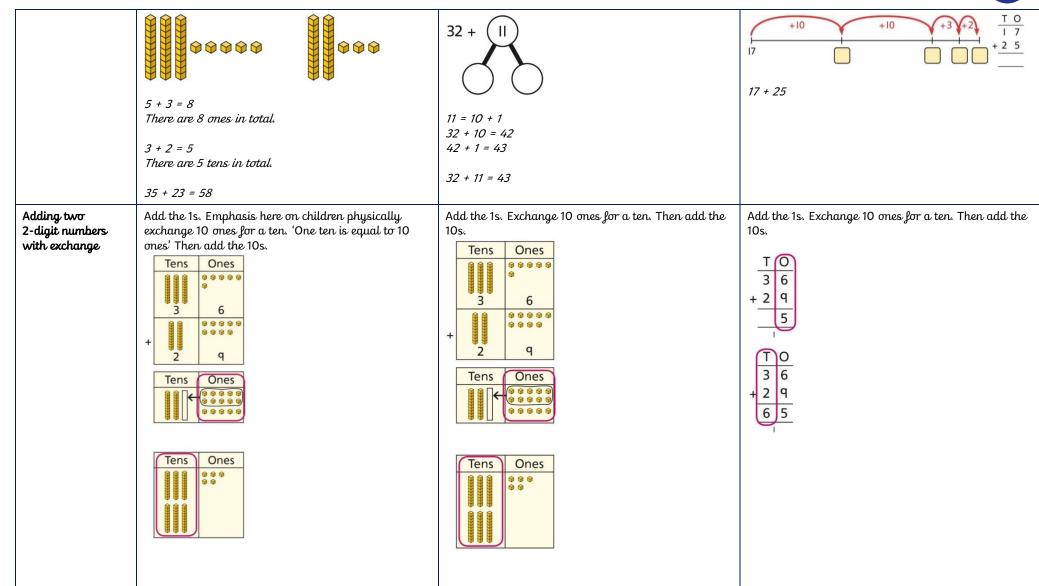


	This can also be done in a place value grid.	T O	This can be represented horizontally or vertically.  34 + 5 = 39
Adding a 1-digit number to a 2-digit number bridging 10	Making 10 Complete a 10 using number bonds.  +	Making 10 Complete a 10 using number bonds.  3 + 9 =	Making 10 Complete a 10 using number bonds.  7 5 2 43 44 45 46 47 48 49 50 51 52 53 7 = 5 + 2 45 + 5 + 2 = 52
Adding a 1-digit number to a 2-digit number using exchange	Making 10 Exchange 10 ones for 1 ten.	Making 10 Exchange 10 ones for 1 ten.	Making 10  Exchange 10 ones for 1 ten.  T O 2 4 8 8 3 2



<del>-</del>		<del>-</del>
Partitioning. Add the 10s and then recombine.	Partitioning Add the 10s and then recombine.	<b>Partitioning</b> Add the 10s and then recombine. $37 + 20 = ?$
	A 100 Square Cut i Support utils tritter starting.  1 2 3 4 5 6 7 8 9 0 10 20 10 10 15 80 7 10 10 12 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	30 + 20 = 50 50 + 7 = 57 37 + 20 = 57
66 is 6 tens and 6 additional ones. 66 + 10 = 76	27 is 2 tens and 7 additional ones. 50 is 5 tens.	
	There are 7 tens in total and 7 additional ones. So, 27 + 50 is 7 tens and 7 additional ones.	
Add the 10s using a place value grid to support.  T O O O O O O O O O O O O O O O O O O	Add the 10s using a place value grid to support.  T O  16 is 1 ten and 6 additional ones. 30 is 3 tens. There are 4 tens and 6 additional ones in total.	Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value.  T O I 6 + 3 O 4 6 $1 + 3 = 4$ $1 \text{ ten } + 3 \text{ tens} = 4 \text{ tens}$ $16 + 30 = 46$
Partitioning Add the 10s and 1s separately.	Partitioning  Add the 10s and 1s separately. Use a part-whole model to support.	Partitioning  Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.
	Add the 10s and then recombine.	Add the 10's and then recombine.  A 100 square can support this understanding.  A 100 square can support this understanding.  66 is 6 tens and 6 additional ones.  66 + 10 = 76  Add the 10's using a place value grid to support.  Add the 10's using a place value grid to support.  Add the 10's using a place value grid to support.  Add the 10's using a place value grid to support.  Add the 10's using a place value grid to support.  To our of tens and 6 additional ones.  30 is 3 tens.  There are 4 tens and 6 additional ones in total.  Partitioning.  Partitioning.



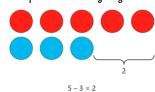




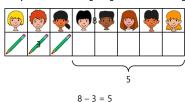
Year 2
Subtraction.

# Subtraction as finding the difference

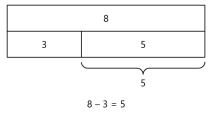
Compare the set of objects in each group



Compare the set of objects in each group

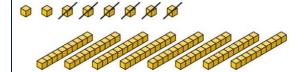


Compare the set of objects in each group

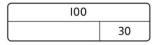


#### Subtracting multiples of 10

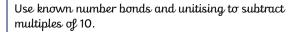
Use known number bonds and unitising to subtract multiples of  $10\,$ .



Use known number bonds and unitising to subtract multiples of  $10\,$ .

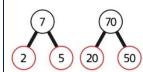


10 - 3 = 7 So, 10 tens subtract 3 tens is 7 tens.



7 tens subtract 5 tens is 2 tens. 70 - 50 = 20

## 8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.



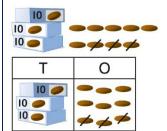
## Subtracting a single-digit number

Subtract the 1s. This may be done in or out of a place value grid.  $\,$ 

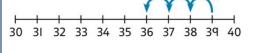


Т	0
	8 8 8 8 9 8 8 9 9

Subtract the 1s. This may be done in or out of a place value grid.  $\ \ \,$ 



Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds.





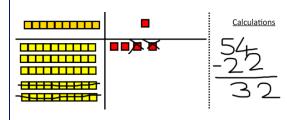


Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.  35 - 6 6 is made of 5 ones plus 1 one. I took away 5 counters, then 1 more.	Bridge 10 by using known bonds.  Drawing representations and crossing out to subtract.  35 - 6  First, I will subtract 5, then 1.	Bridge 10 by using known bonds.  -4  16 17 18 19 20 21 22 23 24 25 26  24 - 6 = ? 24 - 4 - 2 = ?
Subtracting a single-digit number using exchange	Exchange 1 ten for 10 ones Emphasis here on children physically exchange 10 ones for a ten. 'One ten is equal to 10 ones.'  This may be done in or out of a place value grid.  T  O  D  D  D  D  D  D  D  D  D  D  D  D	Exchange 1 ten for 10 ones.  T  O  O  O  O  O  O  O  O  O  O  O  O	Exchange 1 ten for 10 ones.  TO 2 5 - 7 8 TO 2 15 - 7 8 25 - 7 = 18
Subtracting a 2-digit number	Subtract by taking away.	Subtract the 1s and the 10s.  This can be represented on a 100 square.    1   2   3   4   5   6   7   8   9   10     11   12   13   14   15   16   17   18   19   20     21   22   23   24   25   26   27   28   29   30     31   32   23   34   45   46   47   48   49   50     51   52   53   54   55   56   57   58   59   60     61   62   63   64   65   66   67   68   69   70     71   72   73   74   75   76   77   78   79   80     81   82   83   84   85   86   75   88   89   90     91   92   93   94   95   96   87   98   89   90     91   92   93   94   95   96   87   98   89   90     91   92   93   94   95   96   87   98   99   90     91   92   93   94   95   96   87   98   99   90     91   92   93   94   95   96   87   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90     91   92   93   94   95   96   97   98   99   90	Subtract the 1s and the 10s.  This can be represented on a number line. $ \begin{array}{cccccccccccccccccccccccccccccccccc$



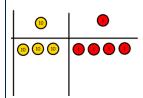
Subtracting a 2-digit number using place value and columns Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid.

36-14=22 T U 30 6 10 4 20 2 Subtract the 1s. Then subtract the 10s.

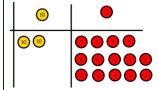


Using column subtraction, subtract the 1s. Then subtract the 10s.

Subtracting a 2-digit number with exchange Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 34-18=



Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones



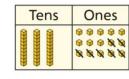
Now I can subtract my ones and my tens.

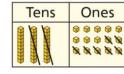
Exchange 1 ten for  $10\,$  ones. Then subtract the 1s. Then subtract the 10s.

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.









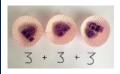
Using column subtraction, exchange 1 ten for  $10\,$  ones. Then subtract the 1s. Then subtract the 10s.



#### Year 2 Multiplication

Equal groups and repeated addition

Recognise equal groups and write as repeated addition and as multiplication.



3 groups of 3 sweets 9 sweets altogether

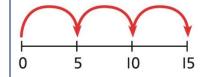
Recognise equal groups using standard objects and write as repeated addition and multiplication.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?





Use a number line and write as repeated addition and as multiplication.



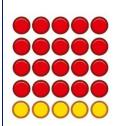
Using arrays to represent multiplication and support understanding Understand the relationship between arrays, multiplication and repeated addition.



4 groups of 6

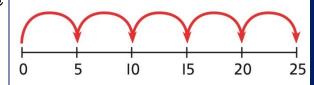
Understand the relationship between arrays, multiplication and repeated addition.

Draw arrays in different rotations to find **commutative** multiplication sentences.



4 groups of 5 ... 5 groups of 5

4×2=8 2×4=8 2×4=8 4×2=8 Understand the relationship between arrays, multiplication and repeated addition.



Understanding commutativity

Use arrays to visualise commutativity.

I can see 6 groups of 3. I can see 3 groups of 6.



Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.



This is 2 groups of 6 and also 6 groups of 2.

Use arrays to visualise commutativity.



Learning ×2, ×5 and ×10 table facts

Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.

3 groups of 10 ... 10, 20, 30 3 × 10 = 30

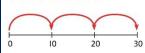


Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.

000000000

000000000

000000000



10 + 10 + 10 = 30 3 × 10 = 30 Understand how the times-tables increase and contain patterns.

10

I × I0 =

10 10

2 × 10 =

10 10 10

3 × 10 =

10 10 10 10

4 × 10 = 5 × 10 =

0 10 10 10 10

6 × 10 =

10 10 10 10 10 10 10

7 × I0 = 8 × I0 =

10 10 10 10 10 10 10 10

9 × 10 =

10 10 10 10 10 10 10 10 10 10

10 × 10 =

10 10 10 10 10 10 10 10 10 10 10

12 × 10 =

5 × 10 = 50 6 × 10 = 60

#### Year 2 Division

Sharing equally

Start with a whole and share into equal parts, one at a time.



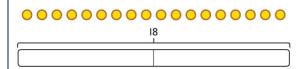


10 shared equally between 2. They get 5 each.

Represent the objects shared into equal parts using a bar model.



20 shared into 5 equal parts. There are 4 in each part. Use a bar model to support understanding of the division.

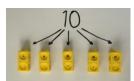


18 ÷ 2 = 9

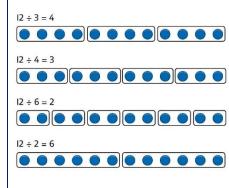


#### Grouping equally

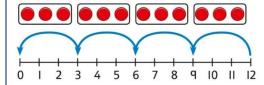
Understand how to make equal groups from a whole.



Understand the relationship between grouping and the division statements.



Understand how to relate division by grouping to repeated subtraction.

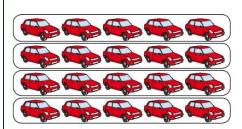


There are 4 groups now.

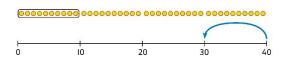
*12 divided into groups of 3. 12 ÷ 3 = 4* 

There are 4 groups.

Using known timestables to solve divisions Understand the relationship between multiplication lacts and division.

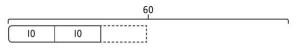


4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5. Link equal grouping with repeated subtraction and known times-table facts to support division.



40 divided by 4 is 10.

Use a bar model to support understanding of the link between times-table knowledge and division.



Relate times-table knowledge directly to division.

$$1 \times 10 = 10$$
  
 $2 \times 10 = 20$   
 $3 \times 10 = 30$   
 $4 \times 10 = 40$   
 $5 \times 10 = 50$   
 $6 \times 10 = 60$   
 $7 \times 10 = 70$   
 $8 \times 10 = 80$   
I used the I0 times-table to help me.  
 $3 \times 10 = 30$ .

I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.

$$3 \times 10 = 30$$
 so  $30 \div 10 = 3$ 



#### **LOWER KEY STAGE 2**

In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns.

By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

**Multiplication and division:** Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.

Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively.

Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit.

Children develop column methods to support multiplications in these cases.

For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.

Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

Fractions: Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find

and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.



	Year 3			
	Concrete	Pictorial	Abstract	
Year 3 Addition				
Understanding 100s	Understand the cardinality of 100, and the link with 10 tens.  Use cubes to place into groups of 10 tens.	Unitise 100 and count in steps of 100.	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.	
Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers.  200 240 241  Use a place value grid to support the structure of numbers to 1,000.  Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.	Represent the parts of numbers to 1,000 using a part-whole model. $ 215 $ $ 200 $ $ 10 $ $ 5 $ $ 215 = 200 + 10 + 5 $	Recognise numbers to 1,000 represented on a number line, including those between intervals.	
Adding 100s	Use known facts and unitising to add multiples of 100.  3 + 4 = 7 3 hundreds + 4 hundreds = 7 hundreds 300 + 400 = 700	Use known facts and unitising to add multiples of 100.  3 + 2 = 5  3 hundreds + 2 hundreds = 5 hundreds  300 + 200 = 500	Use known facts and unitising to add multiples of 100.  Represent the addition on a number line.  Use a part-whole model to support unitising. $3 + 2 = 5$ $300 + 200 = 500$	

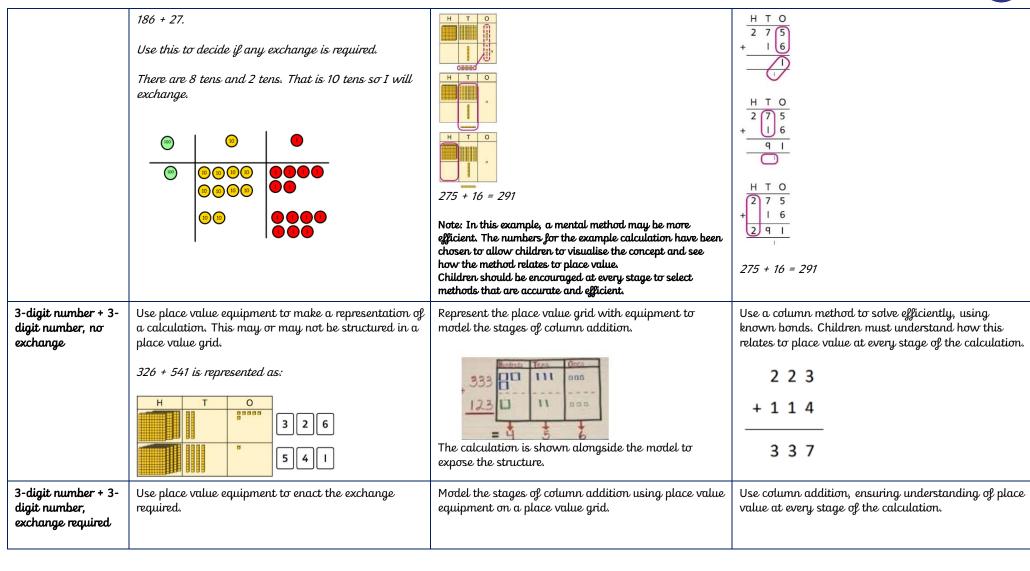


3-digit number + 1s, no exchange or bridging	Use number bonds to add the 1s.  H	Use number bonds to add the 1s.  245 + 4  245 246 247 248 249 250	Understand the link with counting on.  Use number bonds to add the 1s and understand that this is more efficient and less prone to error. $245 + 4 = ?$ I will add the 1s. $5 + 4 = 9$ $S\sigma$ , $245 + 4 = 249$
3-digit number + 1s with exchange	Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten.  Children should explore this using unitised objects or physical apparatus.	Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.  H T O O O O O O O O O O O O O O O O O O	Understand how to bridge by partitioning to the 1s to make the next 10.  135 + 7 = ? 135 + 5 + 2 = 142  Ensure that children understand how to add 1s bridging a 100.  198 + 5 = ? 198 + 2 + 3 = 203
3-digit number + 10s, no exchange	Calculate mentally by forming the number bond for the 10s.	135 + 7 = 142  Calculate mentally by forming the number bond for the 10s. $351 + 30 = ?$	Calculate mentally by forming the number bond for the 10s.  753 + 40

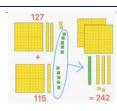


	351 + 30 = ? 5 tens + 3 tens = 8 tens 351 + 30 = 381	5 tens + 3 tens = 8 tens 351 + 30 = 381	I know that 5 + 4 = 9  Sσ, 50 + 40 = 90  753 + 40 = 793
3-digit number + 10s, with exchange	Understand the exchange of 10 tens for 1 hundred.	Add by exchanging 10 tens for 1 hundred.  184 + 20 = ?  H T O O O O O O O O O O O O O O O O O O	Understand how the addition relates to counting on in 10s across 100. $184 + 20 = ?$ I can count in 10s 194 204 $184 + 20 = 204$ Use number bonds within 20 to support efficient mental calculations. $385 + 50$ There are 8 tens and 5 tens. $7hat is 13 tens.$ $385 + 50 = 300 + 130 + 5$ $385 + 50 = 435$
3-digit number + 2- digit number	Use place value equipment to make and combine groups to model addition.	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.
3-digit number + 2- digit number, exchange required	Use place value equipment to model addition and understand where exchange is required.  Use place value counters to represent	Represent the required exchange on a place value grid. $275 + 16 = ?$	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.

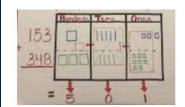








There are 13 ones. I will exchange 10 ones for 1 ten.



126 + 217 = 343

Note: Children should also study examples where exchange is required in more than one column, for example 185 + 318 = ?

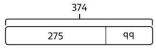
#### Representing addition problems, and selecting appropriate methods

Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps.

These representations will help them to select appropriate methods.

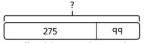
Children understand and create bar models to represent addition problems.

275 + 99 = ?

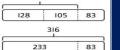


275 + 99 = 374

Use representations to support choices of appropriate methods.  $\begin{tabular}{ll} \end{tabular}$ 



I will add 100, then subtract 1 to find the solution.



128 + 105 = 233

#### Year 3 Subtraction

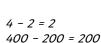
#### Subtracting 100s

Use known facts and unitising to subtract multiples of 100. Expose the structure alongside pictorial representations.

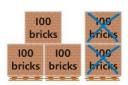








Use known facts and unitising to subtract multiples of 100.



Understand the link with counting back in 100s.



Use known facts and unitising as efficient and accurate methods.

I know that 7 - 4 = 3. Therefore, I know that 700 - 400 = 300.

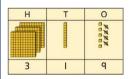


#### 3-digit number – 1s, no exchange

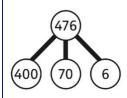
Use number bonds to subtract the 1s.

Н	Т	0
		0000
3	1	q

$$319 - 4 = ?$$



Use number bonds to subtract the 1s.



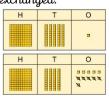
Understand the link with counting back using a number line.

Use known number bonds to calculate mentally.

#### 3-digit number – 1s, exchange or bridging required

Understand why an exchange is necessary by exploring why 1 ten must be exchanged.

Use place value equipment.



Represent the required exchange on a place value grid.

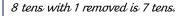
Н	Т	0
0	00000	000000
		00000
		000000

Calculate mentally by using known bonds.

#### 3-digit number – 10s, no exchange

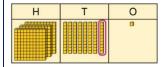
Subtract the 10s using known bonds.

381 - 10 = ?





Subtract the 10s using known bonds.



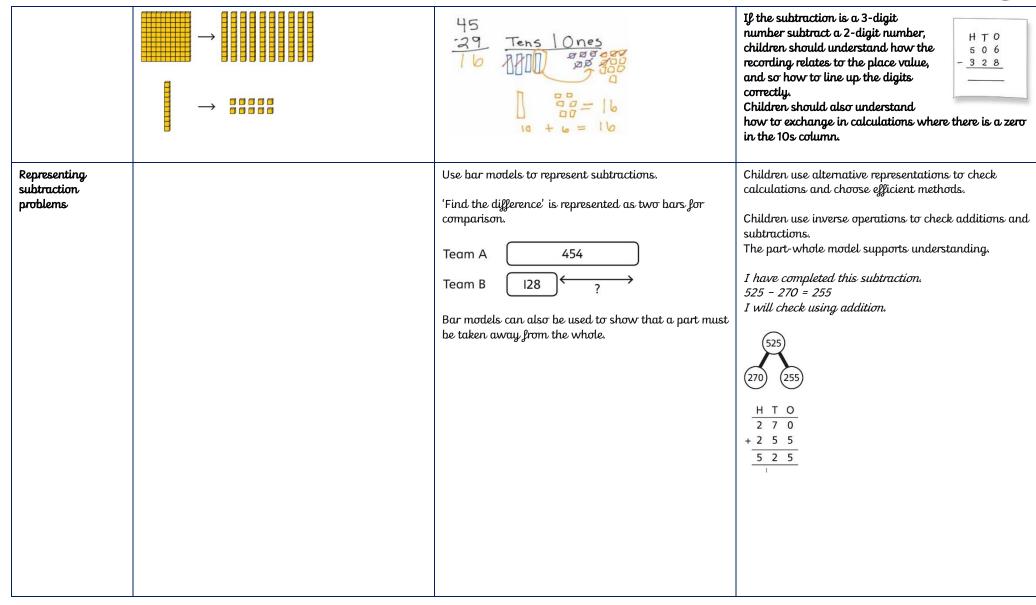
Use known bonds to subtract the 10s mentally.

$$S\sigma$$
,  $372 - 50 = 322$ 



			They act of
3-digit number – 10s, exchange or	Use equipment to understand the exchange of 1 hundred for 10 tens.	Represent the exchange on a place value grid using equipment.	Understand the link with counting back on a number line.
bridging required		I need tσ exchange 1 hundred for 10 tens, tσ help subtract 2 tens.	Use flexible partitioning to support the calculation. $235 - 60 = ?$ $235 = 100 + 130 + 5$ $235 - 60 = 100 + 70 + 5$ $= 175$
		210 - 20 = 190	
3-digit number – up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away.	Represent the calculation on a place value grid.  Calculations 176-64= 176 64 112	Use column subtraction to calculate accurately and efficiently.  H T O 9 9 9 - 3 5 2 - 7 H T O 9 9 9 - 3 5 2 - 4 7 H T O 9 9 9 - 3 5 2 - 6 4 7
3-digit number – up to 3-digit number, exchange required	Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.	Model the required exchange on a place value grid.  175 - 38 = ? I need to subtract 8 ones, so I will exchange a ten for 10 ones.	Use column subtraction to work accurately and efficiently $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$







#### Year 3 Multiplication

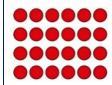
Using commutativity to support understanding of the times-tables

Understand how to use times-tables facts flexibly.





There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. I can use  $6 \times 4 = 24$  to work out both totals. Understand how times-table facts relate to commutativity.



 $6 \times 4 = 24$  $4 \times 6 = 24$  Understand how times-table facts relate to commutativity.

I need to work out 4 groups of 7.

I know that  $7 \times 4 = 28$ 

so, I know that

4 groups of 7 = 28 and 7 groups of 4 = 28.

Understanding and using ×3, ×2, ×4 and ×8 tables.

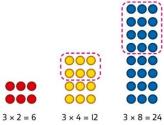
Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.



TARRA III

I can use the ×3 table to work out how many keys. I can also use the ×3 table to work out how many batteries.

Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.



Children understand the relationship between related multiplication and division facts in known timestables.





 $2 \times 5 = 10$  $5 \times 2 = 10$  $10 \div 5 = 2$  $10 \div 2 = 5$ 

Multiplying a 2-digit number by a 1-digit number

Understand how to link partitioning a 2-digit number with multiplying.

Each person has 23 flowers. Each person has 2 tens and 3 ones.

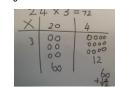






Use place value to support how partitioning is linked with multiplying by a 2-digit number.

$$3 \times 24 = ?$$
  
 $3 \times 4 = 12$   
 $3 \times 20 = 60$   
 $60 + 12 = 72$   
 $3 \times 24 = 72$ 



Use addition to complete multiplications of 2-digit numbers by a 1-digit number.



There are 3 groups of 2 tens. There are 3 groups of 3 ones.

Use place value equipment to model the multiplication context alongside the representation.

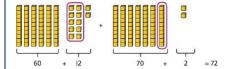
х	Т	U

There are 4 groups of 3 ones.

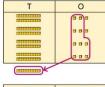
There are 4 groups of 1 tens.

Multiplying a 2-digit number by a 1-digit number, expanded column method Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.

$$3 \times 20 = 60$$



Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.



Т	0

Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.

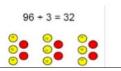


Year 3
Division

Using times-tab	les
knowledge to di	ivide

Use knowledge of known times-tables to calculate divisions.

24 divided into groups of 6 = 4



Use knowledge of known times-tables to calculate divisions.

I need to work out 30 shared between 5.

I know that  $6 \times 5 = 30$ so I know that  $30 \div 5 = 6$ .

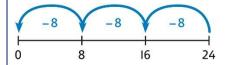
A bar model may represent the relationship between sharing and grouping.

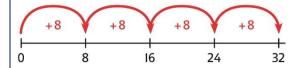
		2	4		
4	4	4	4	4	4

$$24 \div 6 = 4$$

Use knowledge of known times-tables to calculate divisions.

Children understand how division is related to both repeated subtraction and repeated addition.





## Understanding remainders

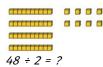
Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.



There are 13 sticks in total.

There are 3 groups of 4, with 1 remainder.

2-digit number divided by 1-digit number, no remainders Children explore dividing 2-digit numbers by using place value equipment.



Use images to explain remainders.



22 ÷ 5 = 4 remainder 2

Children explore which partitions support particular divisions.



Understand that the remainder is what cannot be shared equally from a set.

$$3 \times 5 = 15$$

Children partition a number into 10s and 1s to divide where appropriate.

Children partition flexibly to divide where appropriate.



			ary Aco
	First divide the 10s.  Then divide the 1s.	I need to partition 42 differently to divide by 3. $ 42 = 30 + 12 $ $ 42 \div 3 = 14 $	42 ÷ 3 = ? 42 = 40 + 2 I need to partition 42 differently to divide by 3. 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 42 ÷ 3 = 14
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder.	Use place value equipment to understand the concept of remainder in division.  37 10 10 10 7  Draw dots and group them to divide an amount and clearly show a remainder.  38 39 10 10 10 7	Partition to divide, understanding the remainder in context.  67 children try to make 5 equal lines.  67 = 50 + 17  50 ÷ 5 = 10  17 ÷ 5 = 3 remainder 2  67 ÷ 5 = 13 remainder 2  There are 13 children in each line and 2 children left out.



	Year 4				
	Concrete	Pictorial	Abstract		
Year 4 Addition					
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.  4 thousands equal 4,000.  1 thousand is 10 hundreds.	Represent numbers using part-part-whole once children understand the relationship between 1,000s and 100s. $5,000 + 60 + 8 = 5,068$	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  Understand and read 4-digit numbers on a number line.		
Column addition with exchange	2,000 + 500 + 40 + 2 = 2,542  Use place value equipment on a place value grid to organise thinking.	Use place value equipment to model required exchanges.	Use a column method to add, including exchanges.		
·	7212-4592-	7 1 5 1	Th H T O   5 5 4		
	Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.	Include examples that exchange in more than one column.	Th H T O    1 5 5 4		
			Include examples that exchange in more than one column.		



Representing additions and checking strategies Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.

1,373 799 574 Th H T O
7 9 9
+ 5 7 4
1 3 7 3

I chose to work out 574 + 800, then subtract 1.

2,999 3,001

This is equivalent to 3,000 + 3,000.

Use rounding and estimating on a number line to check the reasonableness of an addition.

0 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 10,000

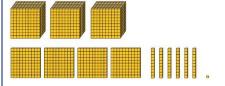
912 + 6,149 = ?

I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.

Year 4 Subtraction

Choosing mental methods where appropriate

Use place value equipment to justify mental methods.



What number will be left if we take away 300?

Use place value grids to support mental methods where appropriate.

Th	Н	Т	0
		8888	00000

7,646 - 40 = 7,606

Use knowledge of place value and unitising to subtract mentally where appropriate.

3,501 - 2,000

3 thousands - 2 thousands = 1 thousand

3,501 - 2,000 = 1,501



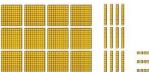
			" Ary Arx
Column subtraction with exchange	Understand why one or more exchanges may be necessary.	Make exchanges across more than one column where there is a zero as a place holder.	Make exchanges across more than one column where there is a zero as a place holder.  The H T O $\frac{\text{Th}}{2} \frac{\text{H}}{48} \frac{\text{T}}{0} \frac{\text{O}}{2}$
	2,502 - 243 = ?  I need to exchange a 10 for some 1s, but there are not any 10s here.	2,502 - 243 = ?	Z,502 - 243 = ?  Th H T O 2 48 9 9 2 - 2 4 3 - 2 4 3 - 2 4 3 - 2 4 3 - 2 4 3 - 2 4 3 - 2 4 3 - 2 5 9
Representing subtractions and checking strategies		Use har models to represent subtractions where a part needs to be calculated.  Total 5.762  ?	Use inverse operations to check subtractions.  I calculated 1,225 - 799 = 574.  I will check by adding the parts. $ \frac{Th H T O}{7 q q} + \frac{5 7 4}{1 3 7 3} $ The parts do not add to make 1,225.  I must have made a mistake.



# Year 4 Multiplication

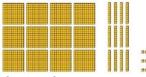
# Multiplying by multiples of 10 and 100

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



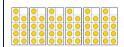
- 3 groups of 4 ones is 12 ones. 3 groups of 4 tens is 12 tens.
- 3 groups of 4 hundreds is 12 hundreds.

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



3 × 4 = 12 3 × 40 = 120 3 × 400 = 1,200

Represent the relationship between the ×9 table and the ×10 table.



Represent the  $\times 11$  table and  $\times 12$  tables in relation to the  $\times 10$  table.



w 11 - 20 + 2





Use known facts and understanding of place value and commutativity to multiply mentally.

$$4 \times 7 = 28$$

# Understanding times-tables up to 12 × 12

Understand the special cases of multiplying by 1 and  $\boldsymbol{\Omega}.$ 



Understand how times-tables relate to counting patterns.

Understand links between the  $\times 3$  table,  $\times 6$  table and  $\times 9$  table  $5 \times 6$  is double  $5 \times 3$ 

 $\times 5$  table and  $\times 6$  table I know that  $7 \times 5 = 35$ so I know that  $7 \times 6 = 35 + 7$ .

×5 table and ×7 table  $3 \times 7 = 3 \times 5 + 3 \times 2$   $3 \times 5 \qquad 3 \times 2$   $3 \times 7 \qquad 3 \times 7$ 

×9 table and ×10 table 6 × 10 = 60 6 × 9 = 60 - 6



Understanding and using partitioning	Make multiplications by partitioning.	Understand how multiplication and partitioning are related through addition.	Use partitioning to multiply 2-digit numbers by a single digit.
in multiplication	4 × 12 is 4 groups of 10 and 4 groups of 2.	4 × 3 = 12 4 × 5 = 20 4 × 8 = 32 4 × 8 = 32 4 × 8 = 32	$18 \times 6 = ?$ $18 \times 6 = 10 \times 6 + 8 \times 6$ $= 60 + 48$ $= 108$
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications.  Make 4 × 136 using equipment.  I can work out how many 1s, 10s and 100s.  There are 4 × 6 ones 24 ones There are 4 × 3 tens 12 tens There are 4 × 1 hundreds 4 hundreds  24 + 120 + 400 = 544	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  3 l 2 3 l 2 3 3 6	Use the formal column method for up to 3-digit numbers multiplied by a single digit.    3
Multiplying more than two numbers	Represent situations by multiplying three numbers together.	Understand that commutativity can be used to multiply in different orders.	Use knowledge of factors to simplify some multiplications. $24 \times 5 = 12 \times 2 \times 5$





Each sheet has 2 × 5 stickers. There are 3 sheets.

There are  $5 \times 2 \times 3$  stickers in total.

$$5 \times 2 \times 3 = 30$$
$$10 \times 3 = 30$$

```
••••• 00000 00000 00000 00000

••••• 00000 00000 00000 00000

•••• 00000 00000 00000 00000
```

$$10 \times 6 \times 2 = 120$$
  
 $60 \times 2 = 120$ 

$$12 \times 2 \times 5 =$$

$$12 \times 10 = 120$$

So, 
$$24 \times 5 = 120$$

#### Year 4 Division

# Dividing multiples of 10 and 100 by a single digit

Use place value equipment to understand how to use unitising to divide.



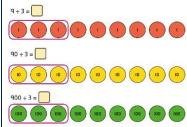
8 ones divided into 2 equal groups 4 ones in each group

8 tens divided into 2 equal groups 4 tens in each group

8 hundreds divided into 2 equal groups

4 hundreds in each group

Represent divisions using place value equipment.



 $9 \div 3 = 3$ 

9 tens divided by 3 is 3 tens. 9 hundreds divided by 3 is 3 hundreds. Use known facts to divide 10s and 100s by a single digit.

$$1500 \div 3 = 500$$

Dividing 2-digit and 3-digit numbers by a single digit by Partition into 10s and 1s to divide where appropriate.

39 ÷ 3 = ?

Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.

39 ÷ 3 = ?

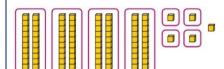
Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate.

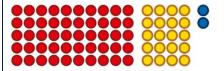


	<del>-</del>		
partitioning into 100s, 10s and 1s	$3 \times 10 = 30$ $3 \times 3 = 9$ $39 = 30 + 9$ $30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$	3 groups of I ten 3 groups of 3 ones $39 = 30 + 9$ $30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed.  42 ÷ 3 = ?  I will split it into 30 and 12, so that I can divide by 3 more easily.	Represent how to partition flexibly where needed.  84 ÷ 7 = ?  I will partition into 70 and 14 because I am dividing by 7. $84 \div 7 = 10$ $14 \div 7 = 2$ $84 \div 7 = 12$	Make decisions about appropriate partitioning based on the division required.  72  72  72  72  72  72  72  72  72  7
Understanding remainders	Use place value equipment to find remainders.  85 shared into 4 equal groups	Represent the remainder as the part that cannot be shared equally.	Understand how partitioning can reveal remainders of divisions.



There are 24, and 1 that cannot be shared.





72 ÷ 5 = 14 remainder 2





#### **UPPER KEY STAGE 2**

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods.

Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

**Multiplication and division:** Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers.

Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000.

Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined.

**Fractions:** Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them.

Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic.

Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.



		Year 5	
	Concrete	Pictorial	Abstract
Year 5 Addition			
Column addition with whole numbers	Use place value equipment to represent additions.  Add a row of counters onto the place value grid to show 15,735 + 4,012.	Represent additions, using place value equipment on a place value grid alongside written methods.   TTh Th H T O O O O O O O O O O O O O O O O O O	Use column addition, including exchanges.    Th Th H T O
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Use approximation to check whether answers are reasonable.  TTh Th H T O
Adding tenths	Link measure with addition of decimals.  Two lengths of fencing are 0.6 m and 0.2 m.  How long are they when added together?	Use a bar model with a number line to add tenths.  0.6 m 0.2 m 0.1	Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$



	0	·6 m		0.	2 m
				ЩЩ	Щ
0.1	0.1	0.1	0.1		
0.1	0.1	0.1	0.1		

0.6 + 0.2 = 0.86 tenths + 2 tenths = 8 tenths 6 tenths + 2 tenths = 8 tenths 0.6 + 0.2 = 0.8

Adding decimals using column addition

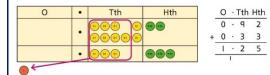
Use place value equipment to represent additions.

Show 0.23 + 0.45 using place value counters.



Use place value equipment on a place value grid to represent additions.

Represent exchange where necessary.



Include examples where the numbers of decimal places are different.

O · Tth Hth

5 · 0 0 + 1 · 2 5 6 · 2 5

0	•	Tth	Hth
00000	•		
	•	<u></u>	00000

Add using a column method, ensuring that children understand the link with place value.

$$\begin{array}{c|cccc}
O & \cdot & \text{Tth Hth} \\
\hline
0 & \cdot & 2 & 3 \\
+ & 0 & \cdot & 4 & 5 \\
\hline
0 & \cdot & 6 & 8
\end{array}$$

Include exchange where required, alongside an understanding of place value.

Include additions where the numbers of decimal places are different.

3.4 + 0.65 = ?



Year 5 Subtraction			
Column subtraction with whole numbers	Use place value equipment to understand where exchanges are required.  2,250 - 1,070	Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. $15,735-2,582=13,153$ The property of the prop	Use column subtraction methods with exchange where required. Drawing attention to accuracy and speed.
Checking strategies and representing subtractions		Bar models represent subtractions in problem contexts, including 'find the difference'.  Athletics Stadium 75,450  Hockey Centre 42,300  Velodrome 15,735 ?	Children can explain the mistake made when the columns have not been ordered correctly.    Bello's working
Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on. $2,002-1,995=?$ Use addition to check subtractions. I calculated $7,546-2,355=5,191$ . I will check using the inverse.



### Subtracting decimals

Use a place value counters to represent the stages of column subtraction, including exchanges where required.

5.74 - 2.25 = ?

0	•	Tth	Hth
00000	•	<u>000000</u>	0000

Exchange I tenth for IO hundredths.

0	•	Tth	Hth
00000	•	9 0 0 0 0 9 Ø	00000 00000 0000

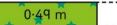
Now subtract the 5 hundredths.

0	•	Tth	Hth
00000		<b>◎ ◎ ◎ ◎ ◎</b>	8888 66668 66666

Now subtract t	he 2	tenths, then t	he 2 ones.
0	•	Tth	Hth
00000		<u></u>	

Use a place value grid to represent the stages of column subtraction, including exchanges where required.

Explore complements to a whole number by working in the context of length.



Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.

$$3.921 - 3.75 = ?$$

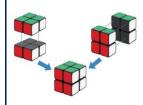
# Year 5 Multiplication

# **Understanding** lactors

Use cubes or counters to explore the meaning of 'square numbers'.

25 is a square number because it is made from 5 rows σf 5.

Use cubes to explore cube numbers.



8 is a cube number.

Use images to explore examples and non-examples of square numbers.



 $8 \times 8 = 64$ 



12 is not a square number, because you cannot multiply a whole number by itself to make 12.

Understand the pattern of square numbers in the multiplication tables.

Use a multiplication grid to circle each square number. Can children spot a pattern?



Multiplying by 10, 100 and 1,000	Use place value equipment to multiply by 10, 100 and 1,000 by unitising. $4 \times I = 4 \text{ ones} = 4$ $4 \times I0 = 4 \text{ tens} = 40$ $4 \times I00 = 4 \text{ hundreds}$ $= 400$	Understand the effect of repeated multiplication by 10.	Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.  H T O T  17 × 10 = 170 17 × 100 = 17 × 10 × 10 = 1,700 17 × 1,000 = 17 × 10 × 10 × 10 = 17,000
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.  5 groups of 3 ones is 15 ones. 5 groups of 3 tens is 15 tens.  So, I know that 5 groups of 3 thousands would be 15 thousands.	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.   1000	Use known facts and unitising to multiply.  5 × 4 = 20 5 × 40 = 200 5 × 400 = 2,000 5 × 4,000 - 20,000  5,000 × 4 = 20,000
Multiplying up to 4- digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. $8 \times 17 = ?$ $8 \times 17 = ?$ $8 \times 10 = 80$ $8 \times 7 = 56$ $80 + 56 = 136$ $80 \times 7 = 136$	Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.  H T O O O O O O O O O O O O O O O O O	Use a column multiplication, including any required exchanges.    3 6



		Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.    ST   ST   ST   ST   ST   ST   ST   S	For children who are not secure with their knowledge of place value, they will use the expanded method:  2741  X 6  6 6 x 1 = 6 (ones x ones)  240 6 x 40 = 240 (ones x tens)  4200 6 x 700 = 4200 (ones x hundreds)  12000 6 × 2000 = 12000 (ones x thousands)  16446  Children should begin by writing down x calculations they have done so mistakes can be easily identified. When accuracy improves these can be left out.
Multiplying up to 4- digits by 2-digits	Manipulatives may still be used with the corresponding long multiplication modelled alongside.  Model Calculation	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Use a column multiplication, including any required exchanges.  2 4  X 1 6  1 4 4  2 4 0  3 8 4
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid. $0.14 \times 10 = 1.4$	Understand how this exchange is represented on a place value chart.



Year 5 Division			
Understanding factors and prime numbers	Use equipment to explore the factors of a given number.	Understand that prime numbers are numbers with exactly two factors.	Understand how to recognise prime and composite numbers.
	24 ÷ 3 = 8 24 ÷ 8 = 3 8 and 3 are factors of 24 because they divide 24 exactly. 24 ÷ 5 = 4 remainder 4. 5 is not a factor of 24 because there is a remainder.	13 ÷ 1 = 13 13 ÷ 2 = 6 r 1 13 ÷ 4 = 4 r 1 1 and 13 are the only factors of 13. 13 is a prime number.	I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.  I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.  I know that 1 is not a prime number, as it has only 1 factor.
Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division.  4,000 $\div$ 1,000  4,000  4,000 is 4 thousands.  4 $\times$ 1,000 = 4,000  So, 4,000 $\div$ 1,000 = 4	Use a bar model to support dividing by unitising.  380 ÷ 10 = 38  7	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.  The Head Tool of The Head
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.	Represent related facts with place value equipment when dividing by unitising.	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.  3,000 $\div$ 5 = 600





15 ones put into groups of 3 ones. There are 5 groups.  $15 \div 3 = 5$ 

15 tens put into groups of 3 tens. There are 5 groups.

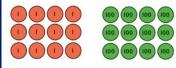
150 ÷ 30 = 5

868668888	

180 is 18 tens.

18 tens divided into groups of 3 tens. There are 6 groups.

180 ÷ 30 = 6



12 ones divided into groups of 4. There are 3 groups.

12 hundreds divided into groups of 4 hundreds. There are 3 groups.

1200 ÷ 400 = 3

3,000	÷	<i>50 =</i>	60
3,000	÷	500	= 6

5 × 600 = 3,000 50 × 60 = 3,000 500 × 6 = 3,000

### Dividing up to four digits by a single digit using short division

Explore grouping using place value equipment.

268 ÷ 2 = ?

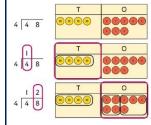
There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones.

264 ÷ 2 = 134

Use place value equipment on a place value grid alongside short division.

The model uses grouping.

A sharing model can also be used, although the model would need adapting.



Use short division for up to 4-digit numbers divided by a single digit.  $\label{eq:digit}$ 

3,892 ÷ 7 = 556

Use multiplication to check.

556 × 7 = ?

 $6 \times 7 = 42$ 



			May Acade
		Lay out the problem as a short division.  There is 1 group of 4 in 4 tens. There are 2 groups of 4 in 8 ones.  Work with divisions that require exchange.  4 9 2 First, lay out the problem.  Problem.  1 0 one of 4 tens with 1 ten left over.  2 groups of 4 tens with 1 ten left over for 10 ones.  We now have 12 ones.  1 0 one of 4 go into 9 tens?  2 groups of 4 tens with 1 ten left over for 10 ones.  We now have 12 ones?  3 groups of 4 ones.	50 × 7 = 350 500 × 7 = 3500 3,500 + 350 + 42 = 3,892
Understanding remainders	Understand remainders using concrete versions of a problem.  80 cakes divided into trays of 6.  80 cakes in total. They make 13 groups of 6, with 2 remaining.	Use short division and understand remainders as the last remaining 1s.  6 8 0	In problem solving contexts, represent divisions including remainders with a bar model.    683



			Than Acade
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange.	Represent division using exchange on a place value grid.	Understand the movement of digits on a place value grid.
1,000	2 ones are 20 tenths.  20 tenths divided by 10 is 2 tenths.	1.5 is 1 one and 5 tenths. This is equivalent to 10 tenths and 50 hundredths. 10 tenths divided by 10 is 1 tenth. 50 hundredths divided by 10 is 1 tenth and 5 hundredths. 1.5 divided by 10 is 1 tenth and 5 hundredths.	0 • Tth Hth Thth 0 • 8 5 0 • 30 38 5  0 • 85 ÷ 10 = 0.085   O • Tth Hth Thth 8 • 5 0 • 0 38 5  8.5 ÷ 100 = 0.085
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division.  1 whole shared between 3 people. Each person receives one-third.	1.5 ÷ 10 = 0.15  Use a bar model and other fraction representations to show the link between fractions and division.  I ÷ $3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$



	Year 6			
	Concrete	Pictorial	Abstract	
Year 6 Addition				
Comparing and selecting efficient methods	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.  M HTh TTh Th H T O	Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation.  Compare written and mental methods alongside place value representations.  The Theory House House and The House House and The House H	Use column addition where mental methods are not efficient. Recognise common errors with column addition.  32,145 + 4,302 = ?  The The Heat Total Street Total St	
Selecting mental methods for larger numbers where appropriate	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.  M HTh TTh Th H T O O O O O O O O O O O O O O O O O O	Use a bar model to support thinking in addition problems.  257,000 + 99,000 = ?  £257,000 £100,000	Use place value and unitising to support mental calculations with larger numbers.  195,000 + 6,000 = ? 195 + 5 + 1 = 201  195 thousands + 6 thousands = 201 thousands So, 195,000 + 6,000 = 201,000	



	This would be 5 more counters in the HTh place.  So, the total is 2,911,301.  2,411,301 + 500,000 = 2,911,301	I added 100 thousands then subtracted 1 thousand.  257 thousands + 100 thousands = 357 thousands  257,000 + 100,000 = 357,000  357,000 - 1,000 = 356,000  Sσ, 257,000 + 99,000 = 356,000	
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results to draw attention to the structure. $3 \times 5 - 2 = ?$	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Understand the correct order of operations in calculations without brackets.  Understand how brackets affect the order of operations in a calculation. $4 + 6 \times 16$ $4 + 96 = 100$ $(4 + 6) \times 16$ $10 \times 16 = 160$
Year 6			

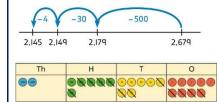
#### Year 6 Subtraction

# Comparing and selecting efficient methods

Use counters on a place value grid to represent subtractions of larger numbers.

	Th	Н	Т	0
•		98888 8	0000X XX	<i>8888</i>

Compare subtraction methods alongside place value representations.



Compare and select methods.

Use column subtraction when mental methods are not efficient.

Use column subtraction for decimal problems, including in the context of measure.



	Th H T O 2 6 7 9 - 5 3 4 2 1 4 5	H T O · Tth Hth 3 0 9 · 6 0
	Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.  computer game  puzzle book  £12-50	$-\frac{2 \ 0 \ 6 \cdot 4 \ 0}{1 \ 0 \ 3 \cdot 2 \ 0}$
	Use a bar model to show how unitising can support mental calculations.  950,000 - 150,000 That is 950 thousands - 150 thousands  950  So, the difference is 800 thousands. 950,000 - 150,000 = 800,000	Subtract efficiently from powers of 10.  10,000 - 500 = ?
Use equipment to explore multiplications alongside the written calculation.  The H T T T T T T T T T T T T T T T T T T	Use place value representations alongside the written method methods where necessary.	Understand short multiplication.  3 2 2 5  ×
4 7	rritten calculation.  The Head of the control of th	If the difference' with two bars as comparison.    Computer game



Multiplying up to a 4-digit number by a 2-digit number		Use an expanded form to expose the structure where necessary.    1	Use compact column multiplication with understanding of place value at all stages.    1 2 3 5     ×
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.  O1	Represent calculations on a place value grid. $3 \times 3 = 9$ $3 \times 0.3 = 0.9$ The parameter of the link between multiplying decimals and repeated addition.	Use known facts to multiply decimals. $4 \times 3 = 12$ $4 \times 0.3 = 1.2$ $4 \times 0.03 = 0.12$ $20 \times 5 = 100$ $20 \times 0.05 = 1$ Find families of facts from a known multiplication.  I know that $18 \times 4 = 72$ .  This can help me work out: $1.8 \times 4 = ?$ $18 \times 0.4 = ?$ $18 \times 0.4 = ?$ $18 \times 0.04 = ?$



Year 6 Division			
Understanding factors	Use equipment to explore different factors of a number.	Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.	Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.
	$24 \div 4 = 6$ $30 \div 4 = 7$ remainder 2 4 is a factor of 24 but is not a factor of 30.	17÷2=8r1	1     (2)     (3)     (4)     (5)     (6)     (7)     (8)     (9)     (10)       (11)     (12)     (3)     (14)     (15)     (16)     (7)     (18)     (9)     (20)       (21)     (22)     (23)     (24)     (25)     (26)     (27)     (28)     (29)     (30)       (31)     (32)     (33)     (34)     (35)     (36)     (37)     (38)     (39)     (40)       (41)     (42)     (43)     (44)     (45)     (46)     (47)     (48)     (49)     (50)
Dividing by a single digit	Use equipment to make groups from a total.  There are 78 in total. There are 6 groups of 13. There are 13 groups of 6.	H T O groups of 6 are in 100?  How many groups of 6 ore in 13 tens?  H T O How many groups of 6 ore in 13 tens?  How many groups of 6 ore in 12 ones?  How many groups of 6 ore in 12 ones?  How many groups of 6 ore in 12 ones?	Use short division to divide by a single digit.  0 6   1   3   2  0   2 6   1   3   2  0   2   2 6   1   3   2
Dividing by a 2- digit number using long division	Use equipment to build numbers from groups.  182 divided into groups of 13. There are 14 groups.	Use bar modelling alongside written division to expose the structure.	Use long division Write the required multiples to support the division process.



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			A slightly different layout may be used, with the division completed above rather than at the side.  3 21 7 9 8 - 6 3 0 1 6 8
			3 8 21 7 9 8 - 6 3 0 1 6 8 - 1 6 8 0  Divisions with a remainder explored in problem-
Dividing decimals	Use place value equipment to explore division of decimals.  O1 O1 O1 O1 O1 O1 O1 O1  8 tenths divided into 4 groups. 2 tenths in each group.	Use a bar model to represent divisions.	solving contexts.  Use short division to divide decimals with up to 2 decimal places.  8 4 · 2 4  0 · 8 4 · <sup>4</sup> 2 4  0 · 5  8 4 · <sup>4</sup> 2 <sup>2</sup> 4  0 · 5 3  8 4 · <sup>4</sup> 2 <sup>2</sup> 4



# <u>Times tables expectations</u>

Towngate Primary Academy believe that when children are proficient in times tables it enables them to calculate more efficiently. Expectations within each year group:

#### Year 1

Children to be able to count in 2's, 10's and 5's

#### Year 2

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts. Summer term introduce 3's.

#### Year 3

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts. Recall the 4, 8 and 3 multiplication tables, and corresponding division facts. Summer term to introduce 6's.

#### Year 4

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts. Recall the 4, 8 and 3 multiplication tables, and corresponding division facts. Recall the 6, 9 and 12 multiplication tables, and corresponding division facts. Recall the 7 and 11 multiplication tables, and corresponding division facts. Statutory multiplication check in Spring term.

#### Year 5 & 6

Rapid recall of all times tables and corresponding division facts through continued practice.